Numerical Simulation of Hydraulic Fracture Propagation using Fully-Coupled Peridynamics, Thin-Film Flow, and Darcian Flow

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Abstract

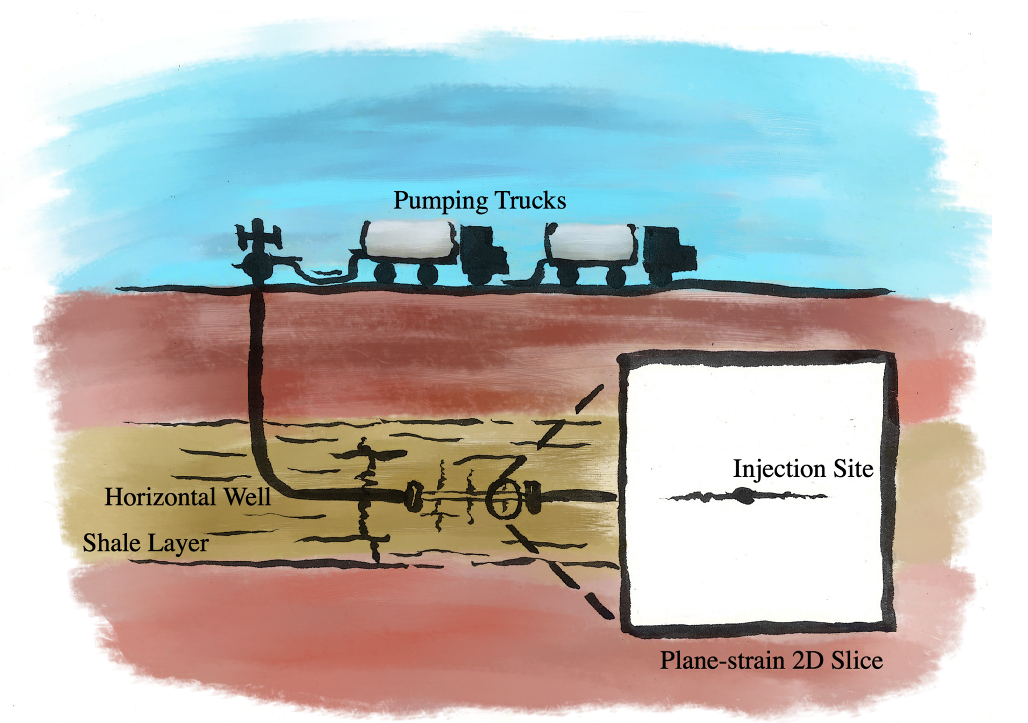
A numerical model is presented for the simulation of the evolution of hydraulic fracture in general geological media that couples a Peridynamic mechanical model and finite element models for porous flow and fracture flow. The two-dimensional model captures porous flow through rock; thin-film flow through hydraulic fractures; mechanical deformation due to applied loads, pore pressure, and fracture pressure; and fracture growth and deformation. The fracture mesh is built dynamically as the fracture grows, connecting broken Peridynamic bonds. While a simple finite element model of Darcian flow is employed in the presented results, the formulation and implementation of the Peridynamic and fracture models allows the code to be easily coupled to any other hydrogeological code. The dynamic evolution of the system is solved by implicit Runge-Kutta integration. The mechanical deformation, matrix pore pressure, and fracture pressure fields are solved fully-coupled in staggered nonlinear iterations at each Runge-Kutta stage, and the damage field is updated sequentially at each time step. The accuracy and convergence rates of the Peridynamic model is studied by comparing numerical results to analytical solutions in linear mechanics, and the fully-coupled model is benchmarked against Terzhaghi's consolidation problem. Applications of the model to simulating pressure-driven hydraulic fracture extension of a lone fracture and a fracture interacting with preexisting natural fractures are presented.

Objectives

Goal: Develop a novel simulator for coupled Geomechanics + Porous flow + Fracture flow to predictively model hydraulic fracture propagation.

1. Explore Peridynamics as a suitable model for fracture propagation:
   (a) Examine three different state-based Peridynamic constitutive responses.
   (b) Determine effect of Peridynamic influence function selection.
   (c) Determine convergence behavior under grid-size refinement
2. Design a new fracture flow model that is strongly coupled to the mechanical deformation and includes leakage into porous rock formation:
   (a) Fracture pressure directly determines material deformation.
   (b) Dynamically mesh the fracture in response to material failure.
   (c) Include mechanical interaction and fluid transfer with natural fracture network.

The model problem guiding method development is hydraulic fracture extension in a horizontal well for gas bearing sediments, but can be applied to other problems, such as thermally-stimulated fracturing in geothermal systems.



Above: Illustration of the model problem. Fluid is pumped along a horizontal well to extend a fracture in a plane-strain slice of the gas-bearing layer.

Peridynamics Formulation

Instead of the continuum mechanics balance of linear momentum PDE, Peridynamics solves the integrodifferential equation at material points in the domain [3]:

$$\rho \ddot{\mathbf{y}} = \int_{\mathcal{H}(x)} \left(\mathbf{t}\left[\mathbf{x}\right]\langle\xi\rangle - \mathbf{t}\left[\mathbf{x}'\right]\langle\xi\rangle \right) \mathrm{d}^3 x' + \rho \mathbf{b}\left(\mathbf{x}\right) \forall \mathbf{x} \in \Omega$$

Discretized material points are connected via bonds with damage factors.

Peridynamics Formulation (continued)

The fracture is initially represented by breaking bonds that cross it. An evolution law is used to dictate the damage response of the material,

$$\alpha\left(t+\Delta t\right) = \min \alpha\left(t\right), f\left(\mathbf{x},\mathbf{y}\right)$$

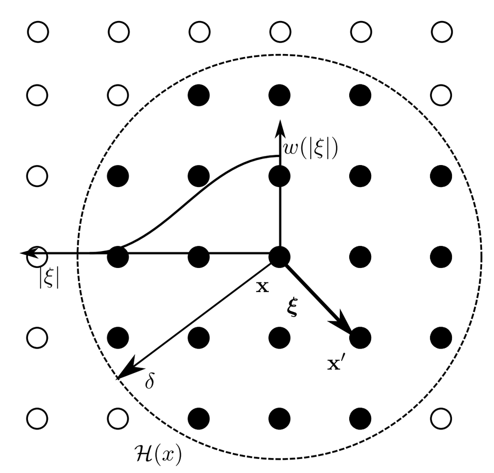
allowing for material points to be completely severed and the fracture to grow. The quantities in the Peridynamic integral are illustrated to the right.

We examined three forms of state-based responses for $\mathbf{t}$: two dilation-based models from Silling, 2007 [3] and Oterkus, 2012 [1], and a deformation gradient-based model from Silling, 2007 [3]. The deformation gradient-based response estimates $\mathbf{F}$ using the positions of the material points and uses it to compute the continuum constitutive response for $\boldsymbol{\sigma}$ to map back to Peridynamic force densities:

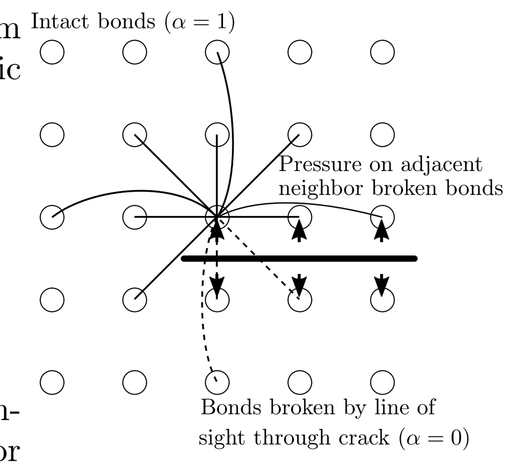
$$\mathbf{F} = \left[\int_{\mathcal{H}} w\alpha \mathbf{Y} \otimes \boldsymbol{\xi} \mathrm{d}x'\right]\left[\int_{\mathcal{H}} w\alpha \boldsymbol{\xi} \otimes \boldsymbol{\xi} \mathrm{d}x'\right]^{-1}$$

$$\mathbf{t}\left[\mathbf{x}\right]\langle\boldsymbol{\xi}\rangle = w\alpha\boldsymbol{\sigma}\left[\int_{\mathcal{H}} w\alpha \boldsymbol{\xi} \otimes \boldsymbol{\xi} \mathrm{d}^n x'\right]^{-1} \boldsymbol{\xi}$$

We solve the discretized system of equations using symbolic differentiation to obtain the consistent tangent for each response.



Peridynamic horizon



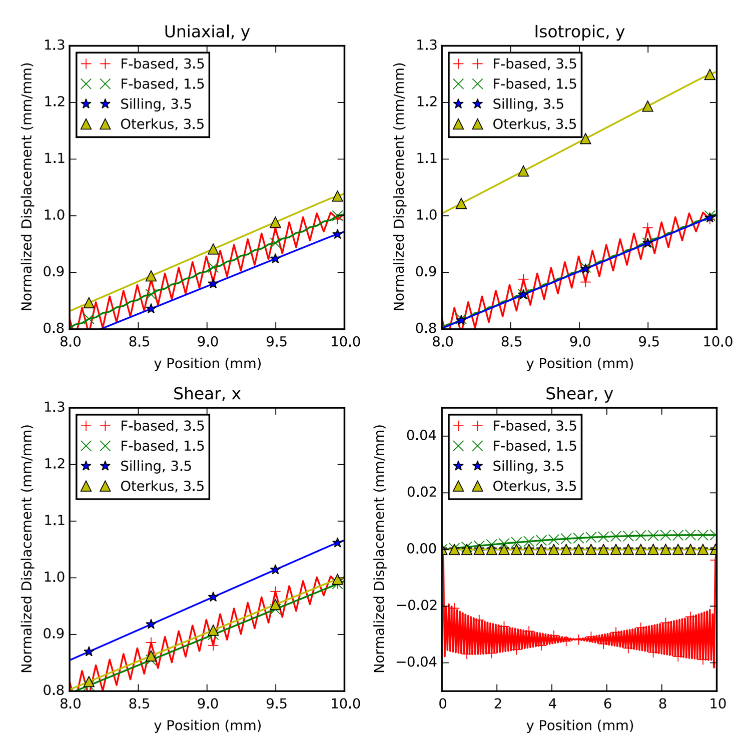
Representation of a fracture

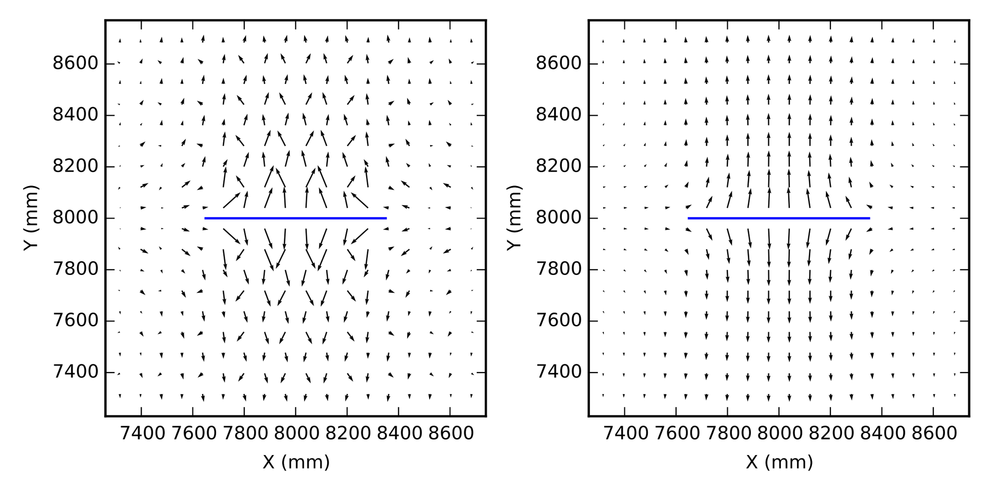
Accuracy and Stability

First, the methods were tested on three constant strain problems on a 10mm by 10mm block: uniaxial compression, isotropic compression, and simple shear. We varied four parameters:

- Response form $\mathbf{t}\left[\mathbf{x}\right]\langle\xi\rangle$.
- Influence function form $w(r)$.
- Grid spacing h
- Relative interaction radius $\mathrm{RF} = \delta/h$

The results using a cubic w with wide interaction radii RF are to the right. Only the F-based model converges for all three cases, but is visibly unstable. The models are also applied to a linearized-fracture mechanics problem of a thin pressurized crack in an infinite domain. The instability in the F-based model is illustrated below. The numerical results achieve less than 5% error from the true solution with 300 by 300 grid after smoothing the solution. Results submitted to Computer Methods in Applied Mechanics and Engineering [2].





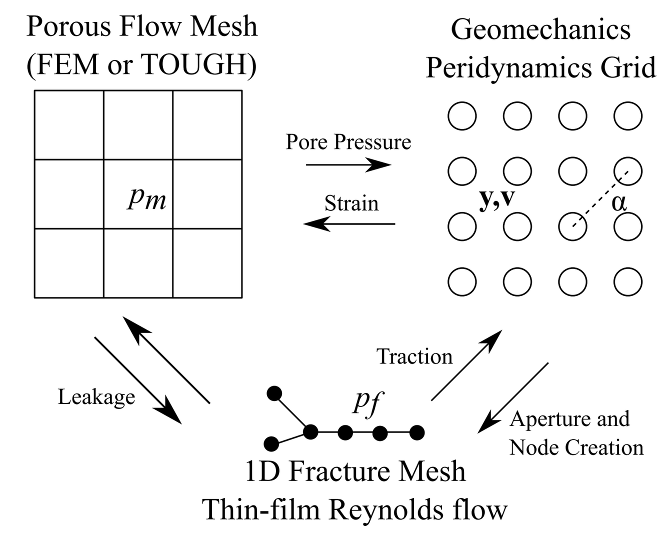
Left, Peridynamics solution of the displacement field around a thin pressurized crack, and, right, the analytical solution. The vectors have a scale of 75× the axes.

Fully Coupled Mechanical+Porous+Fracture Model

The Peridynamics model is coupled to a porous flow solver and a fracture flow solver, resulting in three different numerical methods, each using a different mesh, illustrated to the left. The fields are interpolated directly from the underlying numerical representations for each of the coupling terms.

Both TOUGH or a simple Darcy's law Finite Element solver can be used to solve porous flow. The porous system using an embedded boundary condition for fluid transfer from the fracture.

The fracture is dynamically meshed as the Peridynamic bonds break during the simulation. The fracture pressure is solved using the Reynolds equation for thin film flow along the 1D FEM mesh joining broken bonds:



$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\underbrace{\frac{\rho h^3}{12\mu}}_{\text{Ef. Perm.}}\frac{\mathrm{d}p_f}{\mathrm{d}s}\right) = \frac{\partial \rho}{\partial t}h + \underbrace{\rho\frac{\partial h}{\partial t}}_{\text{Aperture Change}} + \underbrace{\gamma\left(p_f - p_m\right)}_{\text{Coupled BC}}.$$

The aperture h is calculated from positions of neighboring Peridynamics nodes. An internal boundary condition from the fluid pressure is applied as a Peridynamic force density to material points immediately next to the fracture.

Solution of Coupled Systems

The differential equations are marched in time together using implicit Runge-Kutta methods. The mechanics, porous, and fracture fields are solved in a staggered iteration within each stage. The damage field is updated sequentially, which dictates the remeshing step for the fracture. The fracture pressure and mechanics were found to be very tightly coupled, yielding convergence problems with a staggered iteration. The two fields are solved in a fully-linearized monolithic Newton's method simultaneously:

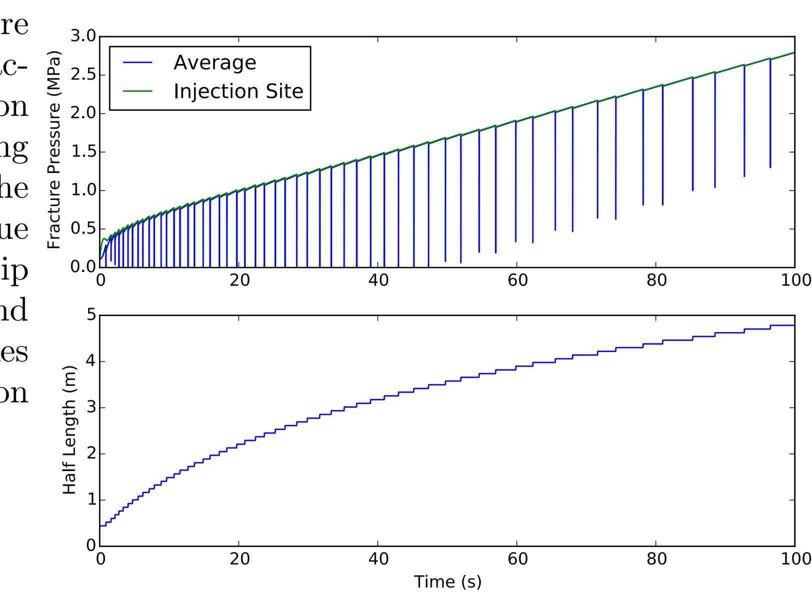
$$\left(\begin{bmatrix}\mathbf{M} & 0\\ 0 & \mathbf{N}\end{bmatrix} + \Delta t\begin{bmatrix}\underbrace{\frac{\partial \mathbf{f}}{\partial \mathbf{v}}}_{\text{Damping}} & \underbrace{\frac{\partial \mathbf{f}}{\partial \mathbf{p}}}_{\text{Traction}}\\ \underbrace{\frac{\partial \mathbf{g}}{\partial \mathbf{v}}}_{\text{Aperture}} & \underbrace{\frac{\partial \mathbf{g}}{\partial \mathbf{p}}}_{\text{Viscosity}}\end{bmatrix} + \Delta t^2\begin{bmatrix}\underbrace{\frac{\partial \mathbf{f}}{\partial \mathbf{y}}}_{\text{Stiffness}} & [\mathbf{0}]\\ \underbrace{\frac{\partial \mathbf{g}}{\partial \mathbf{y}}}_{\text{Aperture}} & [\mathbf{0}]\end{bmatrix}\right)\left\{\begin{matrix}\Delta \mathbf{v}\\ \Delta \mathbf{p}_f\end{matrix}\right\} = \left\{\begin{matrix}\mathbf{R}_v\\ \mathbf{R}_p\end{matrix}\right\}$$

The following algorithm is used to advance the system to the next step:

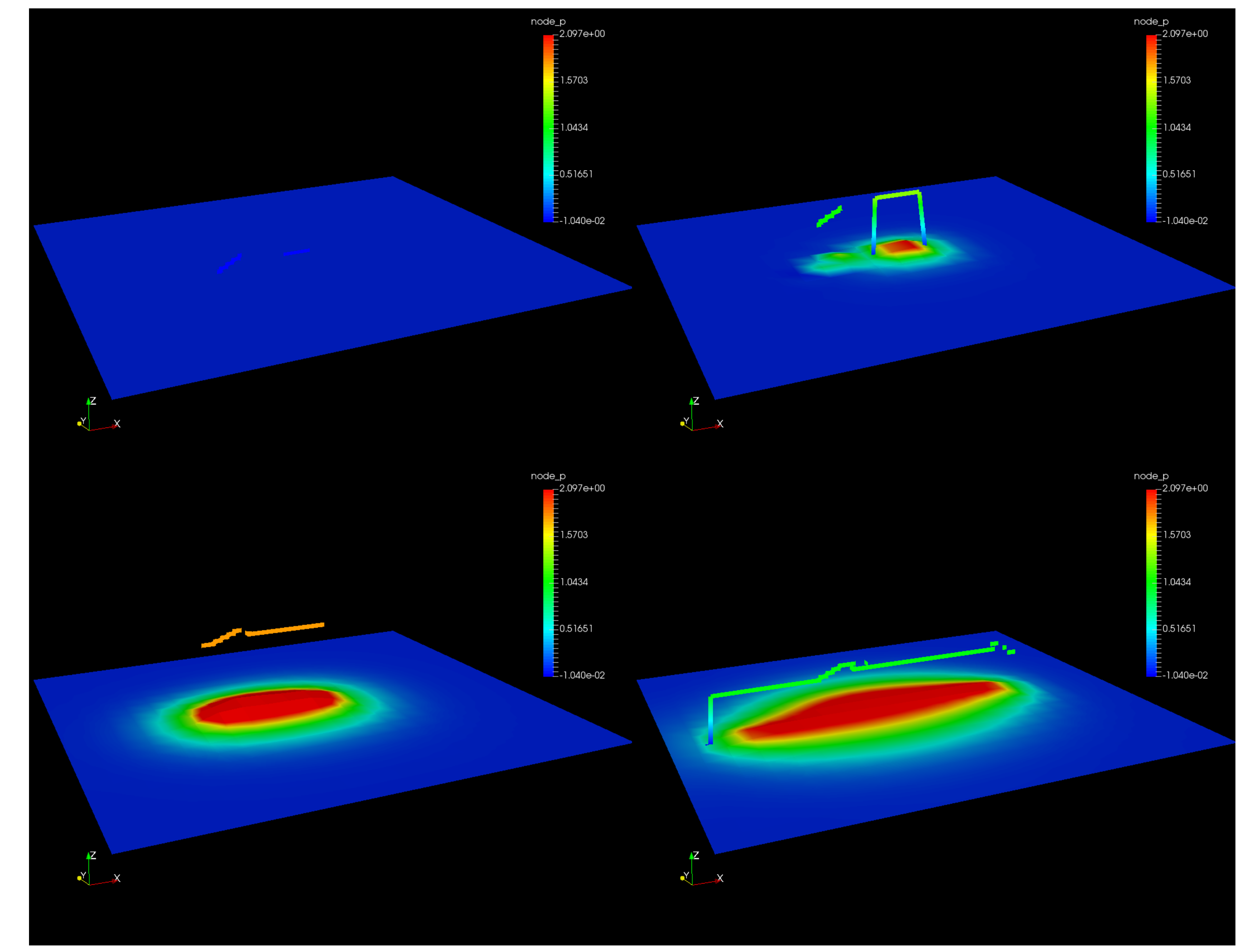
1. Loop over Runge-Kutta stages:
   (a) Staggered iteration until convergence:
      i. Solve mechanics + fracture pressure with Newton's method
      ii. Solve porous flow (linear problem)
2. Update damage $\alpha\left(t+\Delta t\right)$ and find newly broken bonds
3. Remesh fracture, recompute mesh-coupling data structure, and map pressure fields

Results: Extension of a Single Fracture

Right: Evolution in a single-fracture system. The stair-casing of the fracture length is due to the extension occurring as discrete bond-breaking events. The downward spikes in the average fracture pressure are due to the pressure at the fracture tip dropping immediately after a bond breaks. The magnitude of the spikes is reduced by lowering the injection rate.



Results: Interaction with a Natural Fracture



Above: Color/surface plot of fracture pressure (thick line) and pore pressure (surface) as a hydraulically extended fracture interacts with a natural fracture. The pressure in the natural fracture begins to increase due to the pore-pressure increase before the hydraulic fracture intersects it. Mechanical fields that determine fracture growth are not shown.

Summary and Conclusions

We made the following conclusions about the numerical model:

- Silling's deformation gradient Peridynamics model is $O(h)$ accurate but is unstable.
- Smoothing is required for damage evaluation in fracture extension.
- Experimentation with unstructured grids in Peridynamics yielded numerical issues.
- Fracture-mechanical coupling requires monolithic solution of the fracture pressure and mechanical deformation to converge.

Further improvements are needed to the model before extending it into three dimensions and applying it to practical problems. Active development focuses on:

- Exploring other meshless methods to replace Peridynamic mechanical model, such as the Reproducing Kernel Particle Method.
- Developing a multiphase fracture flow solver for proppant transport using a dynamically generated volumetric mesh and discrete particle simulation.
- Coupling the fracture flow field model to TOUGH to interact with practical multiphase reservoir problems.

References

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